

Johann Rafelski\* and Jean Letessier  
*Department of Physics, University of Arizona, Tucson, AZ 85721*  
*and*  
*Laboratoire de Physique Théorique et Hautes Energies\*\**  
*Université Paris 7, 2 place Jussieu, F-75251 Cedex 05.*  
 (June 16, 2000)

We formulate and study the mechanical instability criterion of dense matter fireballs without considering a specific equation of state (EoS). We demonstrate the consistency with the chemical freeze-out of a fireball of matter formed in 158A GeV Pb–Pb collisions. Assuming EoS appropriate for quark-gluon matter, we demonstrate the required deep QGP supercooling prior to sudden hadronization. In a model independent approach, but using results of hadron abundance analysis and lattice QCD, we show that the latent heat of the deconfined phase is bounded from below  $0.14 \text{ GeV/fm}^3 \leq \mathcal{B}$ .

PACS: 12.38.Mh, 25.75.-q, 12.40.Ee,

Hot and dense hadron matter fireball is formed in central collisions of relativistic heavy nuclei, comprising possibly a new state of matter [1]. Driven by internal pressure a fireball expands and ultimately a breakup (hadronization) into final state particles occurs. The mechanism of hadronization and of final hadronic particle abundances formation (chemical freeze-out) are not fully understood [2,3]. However, there is evidence that the fireball breakup occurs suddenly, *i.e.*, over a relatively short period of time [4–7]. It has been in particular pointed out that sudden breakup would arise if a new form of matter fireball significantly supercools and in this state encounters a strong mechanical instability [6].

The more precise formulation of the mechanical instability, which we present here, allows us to obtain a constraint between statistical and physical freeze-out properties of the final state hadrons. We show consistency of this constraint with the results we found in the analysis of particle production in Pb–Pb 158A GeV collisions [8]. We then show how the particle production analysis results can be used to explore the properties of the fireball made of quarks and gluons and derive a limit on the vacuum pressure  $\mathcal{B}$ .

Csörgő and Csernai observed that a rapidly expanding fireball will experience strong mechanical instability when the total pressure is negative [6], at which point the fireball matter cannot fill the available volume, or/and fast spontaneous clustering (*e.g.*, quark-gluon fragmentation-recombination) occurs. They associated this process with deeply supercooled state of matter.

In order to quantify this observation in more general terms we consider the trace of the energy-stress tensor of matter subject to a ‘diluting’ expansion with a local velocity:  $v_c^2 \equiv v_z^2 + v_\perp^2$ :

$$3\mathcal{P} \equiv \text{Tr } T_{kl} = 3P^i + (P^i + \varepsilon^i) \frac{v_c^2}{1 - v_c^2}. \quad (1)$$

The upper index  $i$  refers to a thermal intrinsic frame of reference, locally at rest for the energy density  $\varepsilon^i$  and

pressure  $P^i$  of matter.  $\mathcal{P}$  is thus the pressure of matter in motion, which must be positive for the fireball expansion to continue. As seen in Eq. (1), when the flow velocity remains large but  $\mathcal{P} \rightarrow 0$ ,  $P^i$  must be negative [6].

For  $\mathcal{P} \rightarrow 0$  at  $v_c \neq 0$ , we have a conflict between the desire of the motion to stop or even reverse, and the the diluting expansion. This can be better understood recalling what lead us to consider the condition  $\mathcal{P} \rightarrow 0$ : we visualize the fireball as made of a quark-gluon liquid confined by an external vacuum pressure  $\mathcal{B}$ . In this case the total pressure and energy comprise the vacuum properties:

$$P = P_p - \mathcal{B}, \quad \varepsilon = \varepsilon_p + \mathcal{B}. \quad (2)$$

Subscript  $p$  denotes a quantity solely related to particle properties. Eq. (1) with  $\mathcal{P} = 0$  thus reads:

$$\mathcal{B} = P_p^i + (P_p^i + \varepsilon_p^i) \frac{1}{3} \frac{v_c^2}{1 - v_c^2}, \quad (3)$$

and it describes the condition where the pressure of the expanding quark-gluon fluid is balanced by the external vacuum pressure.

Expansion beyond this point is not possible. A fireball that reached it, either:

- a)** must have  $v_c = 0$  which allows the expansion phase to be followed by contraction — this condition could arise if the initial conditions established in the nuclear interaction are appropriate;
- b)** or it will be torn apart by outward motion at negative pressure, as described above.

We conclude that the condition  $\mathcal{P} = 0$  is the final and definitive instability condition of an expanding hadron matter fireball. The disintegration of fireball matter will be very rapid should a fireball made of a ‘new form of’ matter be at this stage significantly supercooled. Supercooling will develop in the course of fireball expansion, since adiabatic cooling by transfer of heat into the flow of matter occurs as long as no operational (on the scale of  $\tau = 2 \cdot 10^{-23} \text{ s}$ ) instabilities are encountered.

Before looking more closely at the consequences of condition  $\mathcal{P} = 0$ , in Eq. (1), we check if indeed the experimental data is consistent with the corollary that the pressure  $P^i$ , at fireball break up, is *negative*, as is required for  $\mathcal{P} \rightarrow 0$  by Eq. (1). Direct measurement of the intrinsic pressure of an exploding fireball is of course not possible, however, the first law of thermodynamics relates the pressure to the entropy density  $\sigma = S/V$ , energy density  $\varepsilon = E/V$ , and baryon density  $\nu_b = b/V$ :

$$P = T\sigma + \mu_b\nu_b - \varepsilon, \quad (4)$$

with  $V$  being the volume,  $T$  the temperature and  $\mu_b$  the baryochemical potential. Dividing by  $\varepsilon$  we obtain:

$$\frac{PV}{E} = \frac{T_h}{E/S} + \frac{\mu_b}{E/b} - 1 \rightarrow -0.2. \quad (5)$$

The value  $-0.2$ , in Eq. (5), follows using the values from the particle production analysis we obtained for the Pb–Pb collision system [8],  $T_h = 0.143 \pm 0.003$  GeV,  $\mu_b = 0.204 \pm 0.006$  GeV,  $E/b = 7.8 \pm 0.5$  GeV,  $S/b = 42 \pm 3$ , but the ratio of ratios is more precise,  $(E/b)/(S/b) = E/S = 0.185 \pm 0.005$  GeV, as many uncertainties cancel. The collective velocity of the expansion is also determined to be  $v_c = 0.54 \pm 0.025$ , in units of  $c$ . These results arise from a study in which the hadron abundances are used to evaluate the statistical parameters such as  $T_h$ , and employing these subsequently to describe fully the properties of all hadronic particles produced, in order to obtain the values  $E/b, S/b$  found in these final state particles.

It is important to note that we can use these results of hadron production analysis in Eq. (5) since baryon number  $b$ , energy  $E$ , and to a large degree, the entropy  $S$  is conserved both in the process of the fireball dissociation into hadrons, and subsequent further evolution to the kinetic (collisional) freeze-out, if different from chemical freeze-out. As expressed by the negative value of the pressure, Eq. (5), which is in magnitude 20% of the energy density, results of data analysis are consistent with the picture of a deeply supercooled, suddenly exploding fireball. This in turn explains the high degree of chemical non-equilibrium found in the analysis. Also explained are unusual values of certain chemical parameters describing particle abundances [7], such as  $\lambda_s \rightarrow 1$ . This value suggest that both strange  $s$  and antistrange  $\bar{s}$  quarks have same size phase space in the new state of matter, as is in the simplest, unbound particle case.

Eq. (1), for  $\mathcal{P} = 0$ , allows yet a much better understanding of the circumstances of the fireball breakup. Employing the first law, Eq. (4), we obtain

$$\frac{\varepsilon^i}{\sigma T_h + \mu_b \nu_b} = 1 + \frac{1}{3} \frac{v_c^2}{1 - v_c^2}, \quad (6)$$

which is written in terms of *intrinsic* properties of the fireball as:

$$\frac{E^i}{S} \frac{S/b}{S/b + \mu_b/T_h} = T_h \left\{ 1 + \frac{1}{3} \frac{v_c^2}{1 - v_c^2} \right\}. \quad (7)$$

In the final step, we substitute using the Lorentz factor  $E^i/S = \sqrt{1 - v_c^2} E/S$ , and obtain an expression referring to the properties observed in CM-laboratory frame:

$$\frac{E}{S} = \left( T_h + \frac{\mu_b}{S/b} \right) \frac{1}{\sqrt{1 - v_c^2}} \left\{ 1 + \frac{1}{3} \frac{v_c^2}{1 - v_c^2} \right\}. \quad (8)$$

To understand our Eq. (8) better, let us look at a very simple case of a gas of ideal relativistic massless particles. The internal energy density is  $\varepsilon_p = aT^4$ , and the pressure,  $P_p = \varepsilon_p/3 = aT^4/3$ . In consistency with the first law the entropy density is:

$$T\sigma = \varepsilon + P - \mu_b\nu_b \rightarrow 4/3 aT^4. \quad (9)$$

For a negligible baryon density  $\nu_b$ , we find:

$$\frac{E}{S} = \frac{3}{4} T_h, \quad \text{ideal gas at rest.} \quad (10)$$

One of the effects interactions can have is to reduce the mobility of particles which, in fact, impacts most the pressure  $P$ , which can be much smaller than that expected for ideal gas. Eq. (9) for  $P \rightarrow 0$  implies that entropy and energy are nearly equal and thus:

$$\frac{E}{S} \simeq T_h, \quad \text{interacting gas at rest.} \quad (11)$$

The meaning of all the terms in Eq. (8) is now clear: in addition to the temperature term, in Eq. (11), we find a small correction allowing for the influence of baryon density, which we had ignored in Eq. (11). We also have the Lorentz factor which transforms energy from the local flowing matter frame to the CM frame, and the last factor in curly brackets describes the effect of the flow velocity  $v_c$  seen in Eq. (1).

Eq. (8) establishes a general constraint characterizing the fireball break-up condition, which we can evaluate for the case of Pb–Pb interactions at 158A GeV. The solid line, in figure 1, shows the behavior of  $v_c(T_h)$  constraint arising from Eq. (8) for the case  $E/S = 0.185 \pm 0.05$  GeV (error range shown by dashed lines). Outside of the region bounded by the solid line (*i.e.*, for greater  $T_h$  and  $v_c$ ), the flow expansion can occur as the internal particle pressure is greater than the confining pressure.

As the fireball matter approaches the mechanical instability boundary, it turns unstable and is torn apart by the conflict between the desire to expand ( $v_c \neq 0$ ) and the sign of the pressure. The dashed nearly horizontal line is the velocity of sound of the interacting quark-gluon liquid, which differs only slightly from  $1/\sqrt{3}$  [9]. The ‘experimental’ cross, in figure 1, shows the result of our hadron production analysis as stated above [8]. Seeing this result we conclude that hadron production (hadronization) occurred at the mechanical instability of supercooled expanding fireball, and without need

for shock wave formation considering that the expansion velocity does not surpass the velocity of sound.

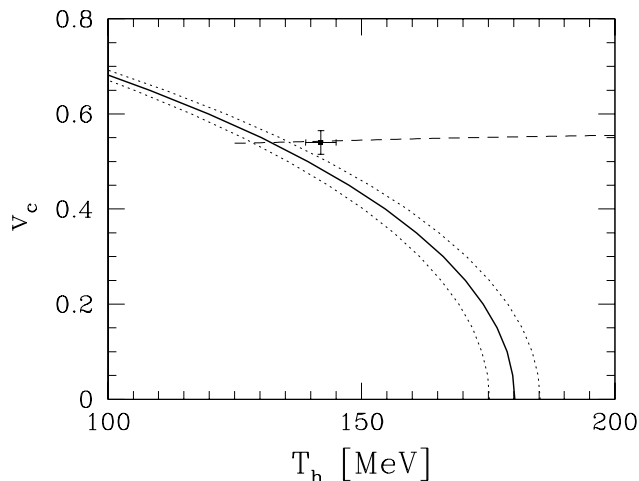


FIG. 1. Fireball velocity as function of break up temperature constraint for the case  $E/S = 0.185 \pm 0.005$  GeV, with  $S/b = 42$  and  $\mu_b = 0.2$  GeV; the dotted lines describe the uncertainty in the determination of  $E/S$ . Dashed line: velocity of sound of relativistic quark-gluon liquid. The experimental point is result of our hadron production analysis.

Our study suggest that mater within an non-homogeneous fireball would hadronize at a single and sharply defined instability condition. We imagine the fireball made of shells of matter, as each of these shells approaches mechanical instability, it hadronizes. Thus the effect of inhomogeneity would be to introduce a finite time associated with particle production, which is much longer than the sudden break-up nature of mechanical instability would suggest.

Up to this point, in the study of instability of a fireball, we have not used any specific properties of the equations of state of the matter filling the fireball. However, our results imply that the matter inside the fireball is deeply supercooled. Can this be a deeply supercooled liquid of quarks and gluons? We have developed a model of QGP equations of state employing properties of QCD interactions and thermal QCD, fine tuned to agree with the properties of lattice QCD results [9]. The parameters that are employed include in particular  $\mathcal{B} = 0.19 \text{ GeV/fm}^3$ .

The thin solid line in figure 2 is the phase boundary in the  $T, \mu_b$  plane, where the pressure of the quark-gluon liquid equals the equilibrated hadron gas pressure. The hadron gas behavior is obtained evaluating and summing the contributions of all known hadronic resonances. The dotted lines, in figure 2, correspond to the condition Eq. (3) for (from right to left)  $v_c^2 = 0, 1/10, 1/6, 1/5, 1/4$  and  $1/3$ . The last dotted line corresponds thus to an expansion flow with the velocity of sound of relativistic noninteracting massless gas. The thick solid line corresponds to an expansion with  $v_c = 0.54$ . The experimental point follows from the analysis of hadron abundances reported above [8].

For vanishing baryon-chemical potential, we note in figure 2 that the phase transition temperature is  $T_p \simeq 173 \text{ MeV}$ . The super-cooled  $\mathcal{P} = 0$  temperature is at  $T_c = 157.5 \text{ MeV}$ , and an expanding fireball can deeply super-cool to  $T \simeq 140 \text{ MeV}$  before onset of the mechanical instability. The scale in temperature we present is result of comparison with lattice gauge results. Within the lattice calculations [10], it arises from the comparison with the string tension.

Comparing thin and thick lines in figure 2 we recognize the deep supercooling as required for the explosive fireball disintegration to occur. With this result we thus have understood how the sudden hadronization mechanism of the QGP-liquid fireball occurs. We have identified this process as a likely evolution stage since our first hadron production data analysis [7].

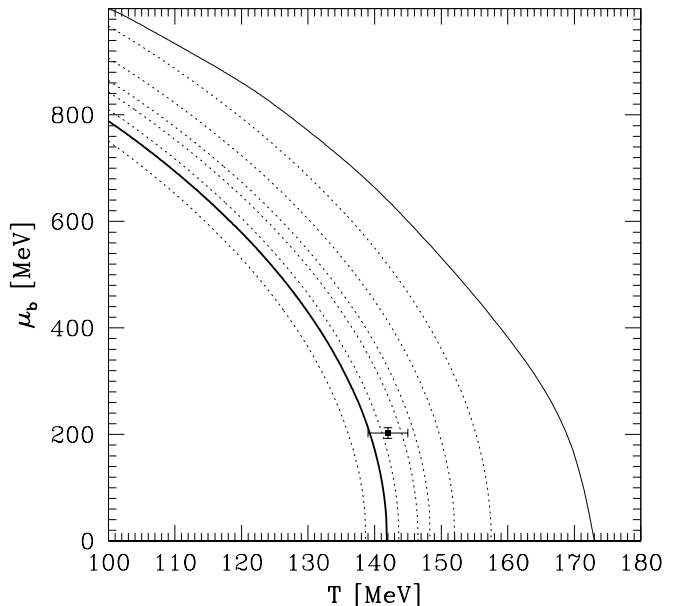


FIG. 2. Thin solid line: Phase transition from hadron gas to QGP liquid. Dotted: breakup condition, Eq. (3), of the liquid for expansion velocity  $v_c^2 = 0, 1/10, 1/6, 1/5, 1/4$  and  $1/3$ , and thick line for  $v_c = 0.54$ . The experimental point denotes chemical freeze-out analysis result [8].

We now consider if we can take advantage of our understanding of the condition of instability to determine in a model independent way the latent heat associated with the formation of the deconfined state of matter. Presence of latent heat, for reasons of physical consistency, requires presence of external (negative) vacuum pressure  $\mathcal{B}$ . More precisely, the vacuum contribution to the physical properties of deconfined matter can be derived from:

$$\ln \mathcal{Z}_{\text{vac}} \equiv -\mathcal{B}V\beta, \quad (12)$$

and we find:

$$P_{\text{vac}} = \frac{T}{V} \ln \mathcal{Z} = -\mathcal{B}, \quad (13)$$

$$\varepsilon_{\text{vac}} = -\frac{\partial \ln \mathcal{Z}}{V \partial \beta} = \mathcal{B} \left\{ 1 + \frac{\partial \ln \mathcal{B}/\mathcal{B}_0}{\partial \ln \beta/\beta_0} \right\}. \quad (14)$$

The temperature,  $T = 1/\beta$ , dependence of the vacuum pressure has been considered within the model of color-magnetic vacuum structure [11,12]. Near to the phase transformation condition the variation of  $\mathcal{B}$  with  $\beta$  is minimal (see figure 2 in [11]), and thus we neglect the logarithmically small last term in Eq. (14).

Reviewing Eq. (5), we see:

$$-\frac{PV}{E} = 0.2 = \sqrt{1 - v_c^2} \frac{\mathcal{B} - P_p^i}{\mathcal{B} + \varepsilon_p^i}, \quad (15)$$

To extract  $\mathcal{B}$  from the measured left hand side of Eq. (15), we need to have a good idea about the properties of the quark-gluon particle gas near to the hadronization point, as well as a ballpark idea about the Lorentz factor connecting the intrinsic rest frame of the fireball with the CM-frame.

A useful inequality follows from Eq. (15):

$$-\frac{PV}{E} \varepsilon = \mathcal{B} - P_p^i < \mathcal{B}. \quad (16)$$

While one would think this is a trivial condition, it can be used in a non-trivial way: the first factor is determined from particle data analysis, and thus the result is, in our case of 158A GeV Pb-Pb collisions,

$$0.2\varepsilon < \mathcal{B}.$$

Next,  $\varepsilon^i$  can be taken from the lattice data and it is well represented by  $\varepsilon^i = aT^4$ , with  $a \simeq 11$ , value extrapolated for the number of light quark flavors being  $n_f = 2/5$  at the hadronization point [9]. Together with the Lorentz factor, which converts from intrinsic to flowing frame of reference, we thus find for the fireball formed in Pb-Pb reactions,

$$0.2 \frac{1}{\sqrt{1 - v_c^2}} 11T_h^4 \simeq 0.14 \text{ GeV/fm}^3 \leq \mathcal{B},$$

which is a rather good limit since the particle pressure  $P_p^i$ , at  $T_h$ , is rather small compared to other terms we consider.

In summary, we have derived a constraint, Eq. (8), which relates the physical parameters of the hadronic fireball at the point of sudden break-up. We obtained this result from mechanical stability consideration employing only general properties of the energy-stress tensor of matter, and the first law of thermodynamics. We showed that this constraint is consistent with analysis results we found considering the experimental particle production results for Pb-Pb collisions at 158A GeV. We have also shown that particle production occurred at condition of significant negative pressure expected in a deeply supercooled state. This demonstrates in a model independent way internal consistency of our (strange) hadron production

analysis [8], since the strong non-equilibrium features we obtained required sudden chemical freeze-out.

We further reported the behavior of the phase transition between hadron gas and quark-gluon liquid, and confirmed the magnitude of the deep supercooling occurring in the fireball expansion. We considered, in a model independent way, the magnitude of the latent heat/vacuum pressure associated with the fireball dynamics. Employing a lattice-QCD based estimate on number of degrees of freedom in the energy density of the QCD thermal matter, we obtained a constraint on the magnitude of latent heat/vacuum pressure  $\mathcal{B} \geq 0.14 \text{ GeV/fm}^3$ .

---

\* Supported in part by a grant from the U.S. Department of Energy, DE-FG03-95ER40937.

\*\* LPTHE, Univ. Paris 6 et 7 is: Unité mixte de Recherche du CNRS, UMR7589.

- [1] <http://www.cern.ch/CERN/Announcements/2000/-NewStateMatter/>. Text of the scientific consensus view of the spokesmen of CERN experiments, also available as E-print: nucl-th/0002042, "Evidence for a New State of Matter: An Assessment of the Results from the CERN Lead Beam Program", compilation by U. Heinz and M. Jacob.
- [2] T.S. Biró, P. Lévai, J. Zimányi, *Phys. Rev. C* **59**, 1574 (1999).
- [3] I.N. Mishustin, *Phys. Rev. Lett.* **82**, 4779 (1999).
- [4] E.E. Zabrodin, L.V. Bravina, L.P. Csernai, H. Stöcker and W. Greiner, *Phys. Lett. B* **423**, 373 (1998).
- [5] Saeed-uddin, *Phys. Lett. B* **406**, 123 (1997).
- [6] T. Csörgő, and L.P. Csernai, *Phys. Lett. B* **333**, 494 (1994).
- [7] J. Rafelski, *Phys. Lett. B* **262**, 333 (1991).
- [8] J. Letessier and J. Rafelski, "Observing Quark-Gluon Plasma with Strange Hadrons", nucl-th/0003014, *Int. J. Mod. Phys. E* (2000) in press; See also: QM99-Torino proceedings, *Nucl. Phys. A* **661**, 497 (1999); Zakopane Summer School Proceedings, *Acta Phys. Polon. B* **30**, 3559 (1999); "Hadrons from Pb-Pb Collisions at 158A GeV, e-Print Archive: nucl-th/9903018 (not accepted by *Phys. Rev. Lett.*); *Advances in Nuclear Dynamics 5*, Kluwer Academic/ Plenum Publishers (New York 1999) W. Bauer and G.D. Westfall, Eds., p. 33, hep-ph/9902365; Strangeness 1998 proceedings, *J. Phys. G* **25**, 295 (1999); and *Acta Phys. Polon. B* **30**, 153 (1999).
- [9] S. Hamieh, J. Letessier and J. Rafelski, "Exploding Quark-Gluon Plasma Fireball", hep-ph/0006085, (June 2000); S. Hamieh, J. Letessier, J. Rafelski, M. Schroedter, and A. Tounsi "Free energy of a hot quark-gluon plasma", hep-ph/0004016 (April 2000).
- [10] F. Karsch, E. Laermann and A. Peikert, *Phys. Lett. B* **478**, 447, (2000).
- [11] B. Müller and J. Rafelski, *Phys. Lett. B* **101**, 111 (1981).
- [12] J.I. Kapusta, *Nucl. Phys. B* **190**, 425 (1981).